

$$U_n = U_{n-1} + 4n + 1$$

$$\begin{aligned} S_n &= \cancel{U_1} + \cancel{U_2} + \cancel{U_3} + \dots + \cancel{U_{n-1}} + U_n \\ &= \underbrace{U_0 + (4 \times 1) + 1}_{= 2} + \underbrace{\cancel{U_1} + (4 \times 2) + 1}_{\dots} + \underbrace{\cancel{U_2} + (4 \times 3) + 1}_{\dots} + \dots + \underbrace{\cancel{U_{n-1}} + 4n + 1}_{\dots} \end{aligned}$$

$$\underbrace{1+1+1+\dots+1}_n = n$$

n fois

$$\begin{aligned} &(4 \times 1) + (4 \times 2) + (4 \times 3) + \dots + 4n \quad \text{Somme des termes suite arithmétique } r=4 \\ &= 4 + 8 + 12 + \dots + 4n \quad \text{dernier} \\ &= \frac{4+4n}{2} \times n \quad \text{nombre de termes} \end{aligned}$$

1^{er} terme

$$\text{Donc } U_n = 2 + \frac{4+4n}{2} \times n + n$$

$$U_n = 2 + (2+2n) \times n + n$$

$$U_n = 2 + 2n + 2n^2 + n$$

$$U_n = 2n^2 + 3n + 2$$